



(To draw/construct the squares, add the square to the base CB and connect the lower corners with the apex of the triangle. Construct perpendiculars as noted and then join their intersections with the sides of the triangle.)

Label the triangle vertices A, B, and C, and by the Law of Cosines calculate the angles of the triangle:

$$A = \arccos \frac{758.595^2 + 869.2^2 - 658^2}{2 \cdot 758.595 \cdot 869.2} = 47^\circ 04' 52.5''$$

$$B = \arccos \frac{869.2^2 + 658^2 - 758.595^2}{2 \cdot 869.2 \cdot 658} = 57^\circ 35' 40.4''$$

$$C = \arccos \frac{658^2 + 758.595^2 - 869.2^2}{2 \cdot 658 \cdot 758.595} = 75^\circ 19' 27.1'', \text{ the sum of which is } 180^\circ 00' 00.0''$$

(Even though only one was needed to calculate the altitude of the triangle, it's always best to compute all three angles as a check)

$$AH = 758.595 \cos 14^\circ 40' 32.9'' = 733.84567 \text{ or } 869.2 \cos 32^\circ 24' 19.6'' = 733.84567 = a$$

The area of triangle ABC equals the area of triangle AGF and trapezoid CGFB, letting CB= b:

$$\frac{b \cdot a}{2} = \frac{(a - S_1) \cdot S_1}{2} + \frac{(b + S_1) \cdot S_1}{2}, \text{ or } b \cdot a = a \cdot S_1 - S_1^2 + b \cdot S_1 + S_1^2 = S_1(b + a)$$

$$\text{so that } S_1 = \frac{b \cdot a}{b + ah}, \text{ and } S_1 = \frac{(658) \cdot (733.84567)}{658 + 733.84567} = 346.928$$

Continuing the same pattern:

$$S_2 = \frac{S_1(a - S_1)}{S_1 + (a - S_1)} = \frac{\frac{b \cdot a}{b + a} \cdot \frac{a^2}{b + a}}{\frac{b \cdot a}{b + a} + \frac{a^2}{b + a}} = \frac{b \cdot a^2}{(b + a)^2},$$

$$\text{making } S_2 = \frac{(658) \cdot (733.84567^2)}{(658 + 733.84567)^2} = 182.917$$

$$\text{Again, continuing the pattern above: } S_n = \frac{b \cdot a^n}{(b + a)^n},$$

so the fifth one,

$$S_5 = \frac{(658) \cdot (733.84567^5)}{(658 + 733.84567)^5} = 26.810$$