



NEW218SOL

Calculate coordinates for points A and B:

A= North 99,607.843, East 97,023.548; B=North 100,549.015, East 99,180.399

Construct circles through points A-P-B and B-Q-C. (These do not need to be to scale)
Draw line P-Q passing through the circles at X and Y. Draw lines A-X, B-X, B-Y & C-Y.
Angle A-P-B = angle A-X-B (because they subtend the same chord from a point on the circumference of the same circle) = $90^{\circ}17'57'' - 43^{\circ}08'42'' = 47^{\circ}09'15''$. Angle X-P-B = angle X-A-B (for the same reason above) = $43^{\circ}08'42''$.

In triangle A-B-X, $A-B = 180^{\circ} - 43^{\circ}08'42'' - 47^{\circ}09'15'' = 89^{\circ}42'03''$. By the Law of Sines:

$$\frac{2353.255}{\sin 47^{\circ}09'15''} = \frac{AX}{\sin 89^{\circ}42'03''} = \frac{BX}{\sin 43^{\circ}08'42''}, \text{ and } AX = 3209.585, BX = 2194.895$$

Angle B-Y-C = angle B-Q-C = $76^{\circ}28'02'' - 49^{\circ}43'02'' = 26^{\circ}45'00''$. Again angle B-Q-Y = angle B-C-Y = $49^{\circ}43'02''$. Angle C-B-Y = $180^{\circ} - 49^{\circ}43'02'' - 26^{\circ}45'00'' = 103^{\circ}31'58''$

By the Law of Sines: $\frac{1429.185}{\sin 26^{\circ}45'00''} = \frac{CY}{\sin 103^{\circ}31'58''} = \frac{BY}{\sin 49^{\circ}43'02''}$, so $CY = 3087.114$

and $BY = 2422.297$. Angle X-B-Y = $360^{\circ} - 89^{\circ}42'03'' - 12^{\circ}23'11'' - 103^{\circ}31'58'' = 44^{\circ}22'48''$

In triangle B-X-Y, $XY^2 + BX^2 - 2 \cdot XY \cdot BX \cdot \cos 44^{\circ}22'48''$ which yields $XY = 1756.784$

By the Law of Sines: $\frac{\sin 44^{\circ}22'48''}{1756.484} = \frac{\sin BXY}{2422.297} = \frac{\sin BYX}{2194.895}$

angle B-X-Y = $74^{\circ}41'42''$ and angle B-Y-X = $60^{\circ}55'30''$
Angle P-X-A = $180^{\circ} - 74^{\circ}41'42'' - 47^{\circ}09'15'' = 98^{\circ}09'03''$
Angle P-A-X = $180^{\circ} - 90^{\circ}17'57'' = 90^{\circ}17'57''$, and by Law of Sines

$$\frac{3209.585}{\sin 90^{\circ}17'57''} = \frac{AP}{\sin 58^{\circ}09'03''} = \frac{PX}{\sin 31^{\circ}33'00''}, \text{ yielding } AP = 2726.388 \text{ \& } PX = 1679.414$$

Angle Q-Y-C = $180^{\circ} - 60^{\circ}55'30'' - 26^{\circ}45'00'' = 92^{\circ}19'30''$, angle Y-C-Q = $180^{\circ} - 92^{\circ}19'30'' - 76^{\circ}28'02'' = 11^{\circ}12'28''$, and by the Law of Sines

$$\frac{3087.114}{\sin 76^{\circ}28'02''} = \frac{YQ}{\sin 11^{\circ}12'28''} = \frac{CQ}{\sin 92^{\circ}19'30''}, \text{ giving } YQ = 617.170 \text{ \& } CQ = 3172.658$$

The bearing of AP = N $66^{\circ}25'31''$ E - $43^{\circ}08'42'' - 31^{\circ}33'00'' = N 8^{\circ}16'11''$ W and point P is North 102,305.882, East 96,631.403

The bearing of CQ = N $55^{\circ}57'40''$ W + $49^{\circ}43'02'' + 11^{\circ}12'28'' = N 4^{\circ}57'50''$ E so that point Q is North 102,909.780, East 100,639.229

(As a check, the inverse distance from P to Q should be $1679.414 + 1756.484 + 617.170 = 4053.067$, which it is)