



New 278 Solution

In this problem triangle DBE is isosceles. Constructing the perpendicular BP gives two $55^{\circ}03'48'' \sim 34^{\circ}56'12'' \sim 90^{\circ}$ triangles with a short side of 206.694, a long side of $206.694 \cdot \tan 55^{\circ}03'12'' = 295.885$ and a hypotenuse of 360.930 for BD and BE.

Let $AD = v$ and $EC = w$, then the tangent of angle A = $295.885/(v+206.694)$ and the tangent of angle C = $295.885/(w+206.694)$.

In any triangle ABC, angle C = $180^{\circ} - (\text{angle A} + \text{angle B})$, therefore $\tan C = \tan[180^{\circ} - (\text{angle A} + \text{angle B})]$ and from the tangent of addition of angles

$$\tan C = \frac{\tan 180^{\circ} - \tan(A+B)}{1 + \tan 180^{\circ} \cdot \tan(A+B)}, \text{ or } \tan C = -\tan(A+B); \tan C = -\frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}.$$

Expanding, $\tan C - \tan A \cdot \tan B \cdot \tan C = -\tan -\tan B$ and $\tan A \cdot \tan B \cdot \tan C = \tan A + \tan B + \tan C$
In this case,

$$\frac{295.885}{v + 206.694} + \frac{295.885}{w + 206.694} + \tan B = \frac{295.885}{v + 206.694} \cdot \frac{295.885}{w + 206.694} \cdot \tan B$$

$$\frac{(295.885)(w + 206.694) + (295.885)(v + 206.694) + (v + 206.694)(w + 206.694) \tan B}{(v + 206.694)(w + 206.694)} = \frac{295.885^2 \cdot \tan B}{(v + 206.694)(w + 206.694)}$$

$$295.885 \cdot w + (295.885)(206.694) + 295.885 \cdot v + (295.885)(206.694) + [v \cdot w + 206.694 \cdot v + 206.694 \cdot w + 206.694^2] \cdot \tan B = 295.885^2 \cdot \tan B$$

$$\text{and, } -137.7772 \cdot w - 137.7772 \cdot v - 2.098087853 \cdot v \cdot w + 216,363.195 = 0$$

Since $v+w = 948.736 - 413.388 = 535.348$, $v = 535.348 - w$, substituting

$$-137.7772 \cdot w - 137.7772(535.348 - w) - 2.098087853(535.348 - w) \cdot w + 216,363.195 = 0$$

which reduces to $2.098087853 \cdot w^2 - 1123.2071 \cdot w + 73,542.3853 = 0$,
and yields $w = 328.342$ and $v = 207.006$

AB and BC are solved using the Law of Cosines:

$$AB = \sqrt{328.342^2 + 360.930^2 - 2 \cdot 328.342 \cdot 360.930 \cdot \cos 124^{\circ}56'12''} = 611.401$$

$$BC = \sqrt{207.006^2 + 360.930^2 - 2 \cdot 207.006 \cdot 360.930 \cdot \cos 124^{\circ}56'12''} = 508.621$$

Angles A and C are solved using the Law of Sines

$$\frac{\sin A}{360.930} = \frac{\sin 124^{\circ}56'12''}{611.401}, \text{ from which angle A} = 28^{\circ}56'36''$$

$$\frac{\sin C}{360.930} = \frac{\sin 124^{\circ}56'12''}{508.621}, \text{ from which angle B} = 35^{\circ}34'23''$$