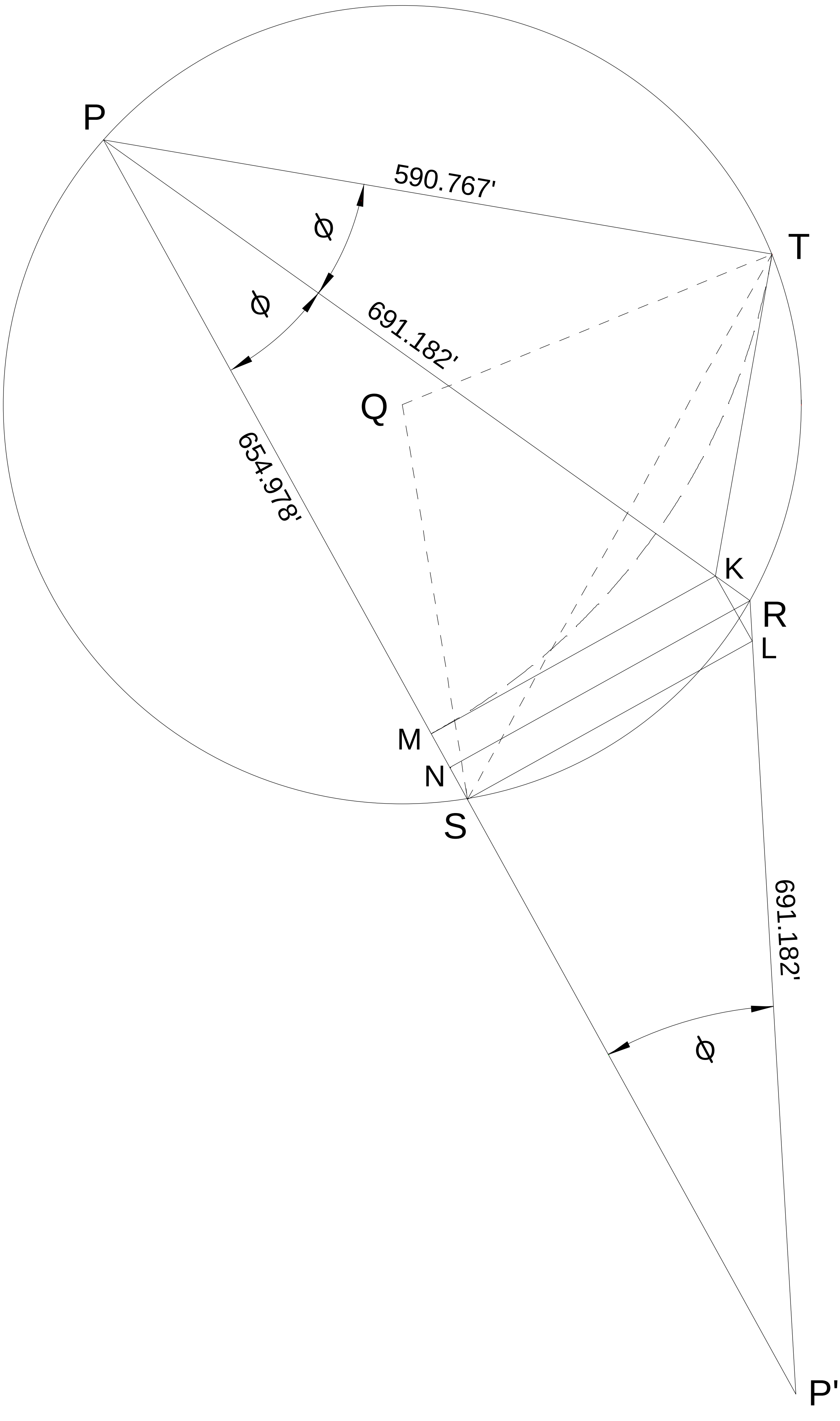
P
590.767'
T
ϕ
ϕ
691.182'
Q
654.978'
K
R
L
M
N
S
691.182'
ϕ
P'

3CHRDANS

Construct RP' = PR = 691.182 to intersect PS extended, making isosceles triangle PRP' with angle RP'S equal to Φ.

Construct PM = PT = 590.767.

MS = PS – PM = 654.978 – 590.767 = 64.211

Construct perpendiculars MK, NR and SL. MN = NS = 32.1055

In triangle PRN, PN = PM+MN = 590.767+32.1055 = 622.8725, and angle Φ equals arccos PN/PR = 622.8725/691.182=25°41'16" and angle TPS is twice that, or 51°22'32"

Draw chord TS. From the Law of Cosines, $PT^2+PS^2-2\cdot PT\cdot PS\cdot \cos P=TS^2$:

$590.767^2+654.978^2-2\cdot 590.767\cdot 654.978\cdot \cos 51°22'32"=TS^2$

And TS =543.081

With Q as the center of the circle, angle TQS is twice angle TPS, or 102°45'04", the central angle of chord TS

The chord TS = 2·R·sin½(102°45'04") = 543.081 from which R =347.570

Ben Remondi Alternate Solution #1 to Dave Lindell's Test Yourself 82 (6/21/22)

Let a=PT; b=PR; c=PS. Central angles to TR & SR are both 2ϕ so that TR=SR. r=radius

Law of Cosines twice: $TR^2 = a^2 + b^2 - 2ab\cos(\phi) = c^2 + b^2 - 2cb\cos(\phi) = SR^2$ Therefore:

$\cos(\phi) = \frac{a+c}{2b}$ and TR=$\sqrt{a^2 + b^2 - 2ab(\frac{a+c}{2b})} = \sqrt{b^2 - ac}$

The central angle to TR is 2ϕ making triangle with sides r & r & TR

Law Cosines: $\text{TR}^2 = r^2 + r^2 - 2rr\cos(2\phi) = 2r^2(1-\cos(2\phi)) = 2r^2 2(1-\cos^2(\phi))$

$$r = \sqrt{\frac{TR^2}{4(1-\cos^2(\phi))}} = \sqrt{\frac{b^2 - ac}{4(1-\cos^2(\phi))}} = 347.5695880317692838516695323181 6$$

Ben Remondi Alternate Solution #2 to Dave Lindell's Test Yourself 82 (6/21/22)

Place X-Y axis at P=(0, 0) where PR is the X axis. a=PT; b=PR; c=PS; C= $\cos(\phi); S=\sin(\phi)$

Equation of all circles radius R centered at (α,β): $R^2=(x-\alpha)^2+(y-\beta)^2$

I: at Point R: (b,0): $R^2=(b-\alpha)^2+(0+\beta)^2=b^2-2b\alpha+\alpha^2+\beta^2$

II: at point P: (0,0): $R^2=\alpha^2+\beta^2$

III: at Point T: (aC,aS): $R^2=(aC-\alpha)^2+(aS-\beta)^2=a^2C^2-2aC\alpha+\alpha^2+a^2S^2-2aS\beta+\beta^2$

IV: at Point S: (cC,-cS): $R^2=(cC-\alpha)^2+(cS+\beta)^2=c^2C^2-2cC\alpha+\alpha^2+c^2S^2+2cS\beta+\beta^2$

II-I: $0=b^2-2b\alpha \quad 0=b-2\alpha \Rightarrow \alpha=\frac{b}{2}$ and Eq. II: $R^2=\frac{b^2}{4}+\beta^2$

$$III-II:\beta=\frac{a-2C\alpha}{2S}=\frac{a-Cb}{2S}$$

$$IV-II:\beta=\frac{Cb-c}{2S}$$

$$\beta=\frac{Cb-c}{2S}=\frac{a-Cb}{2S}\Rightarrow Cb-c=a-Cb \text{ and } C=\frac{a+c}{2b}$$

$$\text{Eq.II}: R=\sqrt{\frac{b^2}{4}+\beta^2}=\sqrt{\frac{b^2}{4}+\frac{(a-Cb)^2}{4(1-C^2)}}=347.56958803176928385166953231816$$

cos(ϕ)=0.90117002468235573264350055412323

$\beta=$ -37.033488136736698701888781641502

$\alpha=345.591$