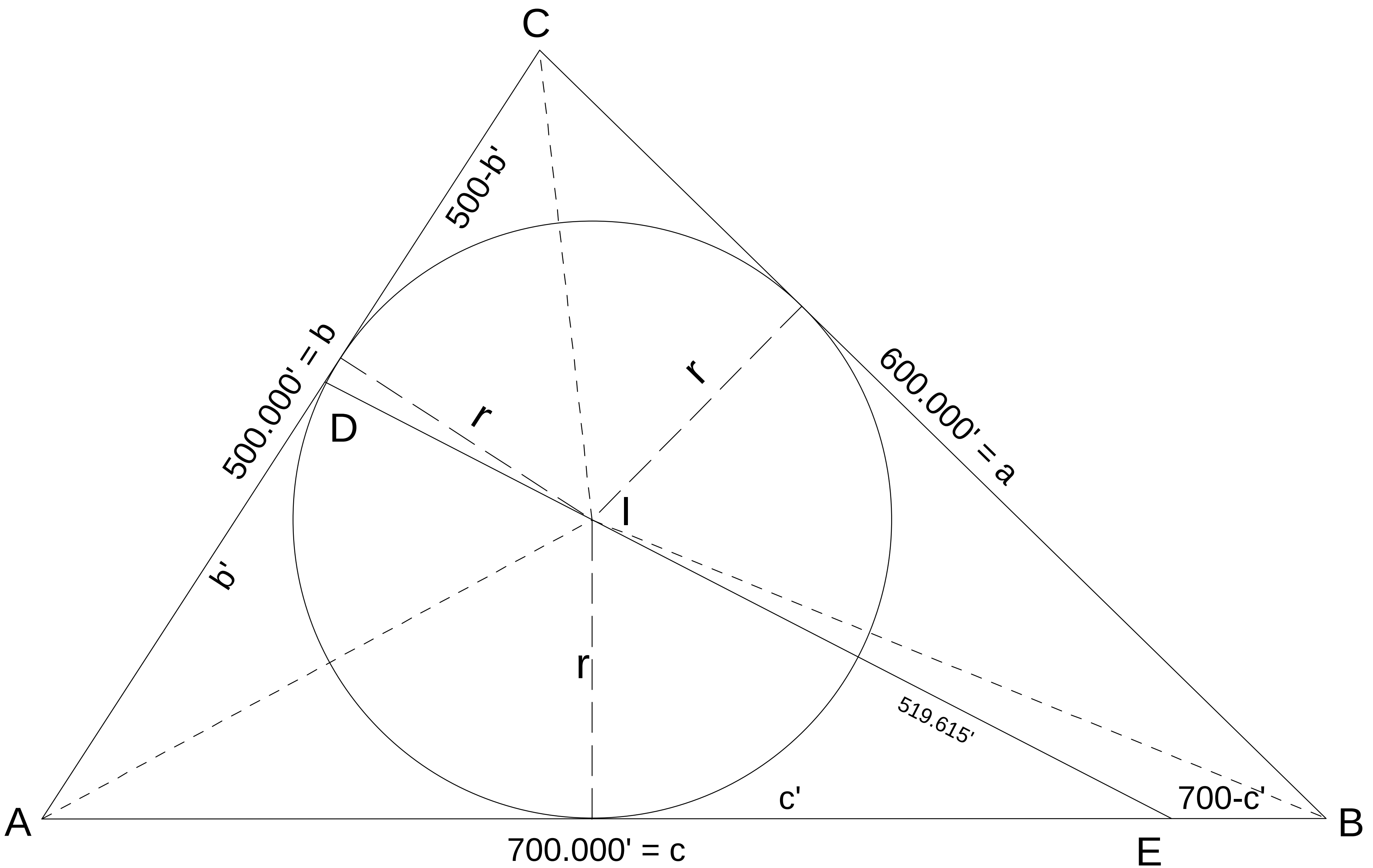
C
500-b'
500.000' = b
D
r
r
I
600.000' = a
b'
r
519.615'
c'
700-c'
A
B
700.000' = c
E

First, hypothesize that line DE actually divides the area of triangle ABC into two equal halves. Let the standard designations for triangles apply, to wit:

Side a is opposite vertex A, b is opposite vertex B , c opposite vertex C.

Let I be the incenter of triangle ABC (the intersection of the angle bisectors and the radius point of the inscribed circle with radius "r").

The area of triangle ABC = twice the area of triangle AED.

Area ABI + area BCI + area CAI = 2x(area AEI + area AID)

$$\frac{c \cdot r}{2} + \frac{a \cdot r}{2} + \frac{b \cdot r}{2} = 2\left(\frac{c' \cdot r}{2} + \frac{b' \cdot r}{2}\right)$$

a + b + c = 2(c' + b')

a + (b - b') + (c – c') = b' + c'

BC + CD + EB = DA + AE, which shows the line also divides the perimeter in half if it passes through the incenter. And the corollary is also true, if the line divides the perimeter in half it also divides the area in half if it passes through the incenter.

The area of triangle ABC = $\frac{1}{2} \cdot 500 \cdot 700 \cdot sinA = 2 \cdot \frac{1}{2} \cdot b' \cdot c' \cdot sinA$

Dividing each side by sinA and 175,000 = b'·c', but b' + c' must equal 900, half the perimeter.

c' = 900 – b', so b'·(900 – b') = 175,000

$b'^2 - 900 \cdot b' + 175{,}000 = 0$, from which b' = 615.831 and 284.169

The only vertex this will fit is A, so AD = 284.169 and AE = 615.831

The length of line DE is then 519.615'

(Note that the total area was not needed to solve this.)