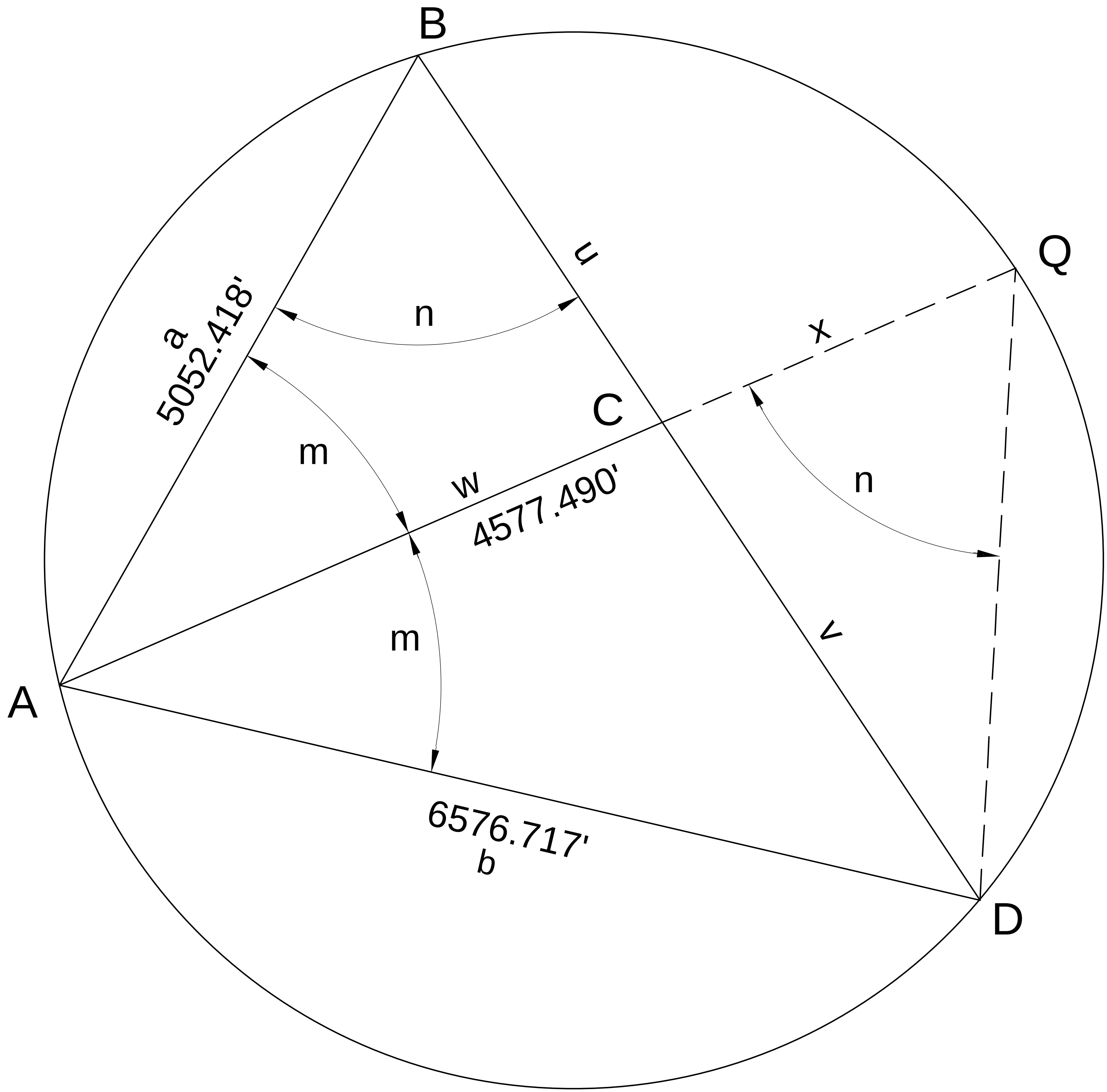
B
Q
u
n
x
a
5052.418'
C
m
n
w
4577.490'
m
v
A
6576.717'
b
D

NEW184ANS

Construct The circumcircle of triangle A-B-D. Extend A-C to intersect the circle at Q. Draw D-Q. Label the parts as shown: A-B = a, A-D = b, A-C = w, C-Q = x, B-C = u, and C-D = v.

Angle B-A-C = angle Q-A-D, (that's a given), angle A-B-D = n = angle A-Q-D because they subtend the same chord from a point on the circumference, and w·x = u·v, because the product of internal chord segments are equal.

Triangles D-A-Q and C-A-B are similar, so $\frac{a}{w} = \frac{w+x}{b}$,

or $a \cdot b = w(w + x) = w^2 + w \cdot x$

$4{,}577.490^2$ = (5,052.418)(6,576.717) –u·v, and u·v = 12,274,908.65...............[1]

By the Law of Cosines, $v^2 = w^2+b^2-2 \cdot w \cdot b \cdot \cos m$ and $u^2 = a^2+w^2-2 \cdot a \cdot w \cdot \cos m$

$$\frac{w^2 + b^2 - v^2}{2 \cdot w \cdot b} = \frac{a^2 + w^2 - u^2}{2 \cdot w \cdot a} = \cos m$$

Multiplying both sides by 2·w·a·b, $a \cdot (w^2+b^2-v^2) = b \cdot (a^2+w^2-u^2)$, expanding, rearranging and substituting from [1] above:

$u^2 = w^2 + a^2 - a \cdot b - \frac{a \cdot w^2}{b} + \frac{12{,}274{,}908.65^2 \cdot a}{b \cdot u^2}$, then multiplying by u^2,

$u^4 = u^2(w^2 + a^2 - a \cdot b - \frac{a \cdot w^2}{b}) + \frac{12{,}274{,}908.65^2 \cdot a}{b \cdot u^2}$ and solving for u^2 by the quadratic equation

$$u^2 = \frac{w^2 + a^2 - a \cdot b - \frac{a \cdot w^2}{b} \pm \sqrt{\left(w^2 + a^2 - a \cdot b - \frac{a \cdot w^2}{b}\right)^2 + 4 \cdot \frac{12274908.65^2 \cdot 5052.418}{6576.717}}}{2}$$

with all the quantities on the right side known

$u^2 = \frac{-2{,}844{,}980.40 \pm \sqrt{2{,}844{,}980.40^2 - 4 \cdot 115{,}751{,}507{,}809{,}455.23}}{2}$

$u^2 = \frac{-2{,}844{,}980.40 \pm 21{,}704{,}836.897}{2}$ = 9,429,928.248 and u =3,070.819

from [1] above: 3,070.819·v = 12,274,908.65 and v = 3,997.275

B-D = u +v = 3,070.819 + 3,997.275 = 7.068.094