



3-ANG-AREA-ANS

Construct the altitude B-B'.

A-B' is equal to $c \cdot \cos 70^\circ 43' 38''$, which is also equal to $c \cdot \sin 19^\circ 16' 22''$.

B'-C is equal to $a \cdot \cos 45^\circ 08' 15''$, which is also equal to $a \cdot \sin 44^\circ 51' 45''$.

The area of A-B-C is the sum of the two right triangles, to wit:

$$\begin{aligned} \text{Area} &= \frac{1}{2} \cdot c \cdot \sin 70^\circ 43' 38'' \cdot c \cdot \sin 19^\circ 16' 22'' + \frac{1}{2} \cdot c \cdot \sin 70^\circ 43' 38'' \cdot a \cdot \sin 44^\circ 51' 45'' \\ &= \frac{1}{2} \cdot c \cdot \sin 70^\circ 43' 38'' \cdot (c \cdot \sin 19^\circ 16' 22'' + a \cdot \sin 44^\circ 51' 45'') \end{aligned}$$

but $c \cdot \sin 19^\circ 16' 22'' + a \cdot \sin 44^\circ 51' 45'' = b$, so area = $\frac{1}{2} \cdot b \cdot c \cdot \sin 70^\circ 43' 38'' \dots [1]$

Now do the same for the other altitudes and you will get the area =

$\frac{1}{2} \cdot a \cdot c \cdot \sin 64^\circ 08' 07'' \dots [2]$ and $\frac{1}{2} \cdot a \cdot b \cdot \sin 45^\circ 08' 15'' \dots [3]$, yielding three equations in three unknowns.

Equating [1] and [3]: $\frac{1}{2} \cdot b \cdot c \cdot \sin 70^\circ 43' 38'' = \frac{1}{2} \cdot a \cdot b \cdot \sin 45^\circ 08' 15''$, then dividing by $b/2$:

$c \cdot \sin 70^\circ 43' 38'' = a \cdot \sin 45^\circ 08' 15''$ so $c = \frac{a \cdot \sin 45^\circ 08' 15''}{\sin 70^\circ 43' 38''}$; substituting in [2]:

$217.946 = \frac{1}{2} \cdot a \cdot \frac{a \cdot \sin 45^\circ 08' 15''}{\sin 70^\circ 43' 38''} \cdot \sin 64^\circ 08' 07''$ yields $a = 803.20$, which substituted into [2] yields $c = 603.11$.

The Law of Sines can be used to solve for $b = 765.65$

Alternatively: also from the Law of Sines, substitute $b = \frac{c \cdot \sin B}{\sin C}$ into [1] for

$$\frac{1}{2} \cdot \frac{c \cdot \sin B}{\sin C} \cdot c \cdot \sin A = \frac{c^2 \cdot \sin B \cdot \sin A}{2 \cdot \sin C} \text{ or } \frac{c^2 \cdot \sin 64^\circ 08' 07'' \cdot \sin 70^\circ 43' 38''}{2 \cdot \sin 45^\circ 08' 15''} = 217,946$$

From which $c = 603.11$

Also area = $\frac{b^2 \cdot \sin A \cdot \sin C}{2 \cdot \sin B}$ from which $b = 765.65$, and area = $\frac{a^2 \cdot \sin B \cdot \sin C}{2 \cdot \sin A}$ from

Which $a = 803.20$