

$r_a \cdot r_b + r_b \cdot r_c + r_c \cdot r_a = s^2$ , where  $r_i$  is the excircle radius opposite side<sub>i</sub>

$$1 \cdot 3 + 3 \cdot 2 + 2 \cdot 1 = 11 \text{ so that } s = \sqrt{11} \dots\dots\dots [1]$$

where  $s = \frac{a+b+c}{2}$  and a, b, c are the sides of the triangle..... [2]

$$r_a = \frac{2A}{c-a+b} \dots\dots\dots [3]$$

$$r_b = \frac{2A}{a-b+c} \dots\dots\dots [4]$$

$$r_c = \frac{2A}{a+b-c}, \text{ where A is the area of the triangle} \dots\dots\dots [5]$$

from [3] with  $r_a$  known to be 1:  $c - a + b = 2A$  ..... [6]

from [4] with  $r_b$  known to be 3:  $3a - 3b + 3c = 2A$ ..... [7]

from [5] with  $r_c$  known to be 2:  $2a + 2b - 2c = 2A$ ..... [8]

equating [6] and [7]:  $c - a + b = 3a - 3b + 3c$  and  $2a = 2b - c$  ..... [9]

equating [6] and [8]:  $c - a + b = 2a + 2b - 2c$  and  $b = 3c - 3a$  ..... [10]

equating [7] and [8]:  $3a - 3b + 3c = 2a + 2b - 2c$  and  $a = 5b - 5c$ ..... [11]

subtracting twice [11] from [9]:  $b = \frac{9c}{8}$  ..... [12]

substituting [12] into [11]:  $a = 5 \cdot \frac{9c}{8} - 5c = \frac{5c}{8}$  ..... [13]

substituting the values from [12] and [13] into [2] and squaring to equal [1]:

$$\left( \frac{\frac{5c}{8} + \frac{9c}{8} + c}{2} \right)^2 = 11, \text{ from which } c = 2.4120908$$

substituting back into [12] and [13],  $a = 1.5075567$  and  $b = 2.713602$